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SPECULATIVE HOLDINGS UNDER LINEAR EXPECTATION PROCESSES-- A MEAN-VARIANCE APPROACH

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Abstract

In this paper, we considered a discrete time abstract market model where the associated commodity is storable. Also, instead of assuming expected profit maximizing speculators, we assumed they employed mean-variance approaches.

Within this framework, given a non-degenerate quadratic inventory cost function and a linear expectation process, the optimal speculative carryover may be decomposed into four components of which two are special features arising from mean-variance considerations.

Furthermore, assuming a linear non-speculative excess demand function, Friedman's conjecture (i.e., profitable speculation necessarily stabilizes prices) holds fron an *ex ante* point of view.

Speculative Holdings Under Linear Expectation Processes—

A Mean-Variance Approach*

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I. Introduction

There have been several studies that attempt to characterize speculators' behavior under linear expectation processes [2] [4]. However, these papers assumed that speculators are expected profit maximizers, regardless of the riskiness of their market operations.[1] In this paper, we assume that speculators employ a mean-variance approach, and then characterize their impacts on the market again assuming a linear expectation process. Within this framework, and assuming a linear non-speculative excess demand function, Friedman's conjecture holds (i.e., profitable speculation necessarily stabilizes prices) from an ex ante point of view.

The plan of this paper is as follows: In section II, we describe the market structure and the speculator's problem; in section III and IV, dynamic programming is applied to solve the speculator's problem and some properties of the solution are exploited. In section V, linear expectation rules are introduced. We consider the special case when inventory cost is a fixed constant in section VI. The general case is dealt with in section VII.

Non-speculative excess demand is introduced in section VIII. Using this, we derive market price behavior and examine Friedman's

conjecture in section IX. Finally, section X states the conclusions.

II. Market Structure

Consider a discrete time spot market, where the associated commodity is storable. There is no forward or futures market in this commodity, and short-selling in the spot market is prohibited. The market opens at time $t = 0, 1, 2, \ldots$ and transactions take place immediately thereafter.

There are three different types of agents in this market: producers, speculators and consumers. Producers and consumers as a group are called non-speculators. The type of each agent is exogeneously determined. We also assume that the decisions of producers and consumers are made without considering the effects of speculators. Hence, we can treat non-speculative excess demand as exogeneously given. Random effects that enter the model either come from the production side or the non-speculative demand side, but are assumed to be independent of speculators' behavior.

Each speculator takes prices as given (i.e., the case of competitive speculation) and he employs a mean-variance approach to solve his decision problem, using all information available to him. Let S_t denote the stock level at time t for a specified speculator (later, we'll assume all speculators are identical). Now, at time t, the speculator observes the market price P_t and his carryover from the previous period S_{t-1}. He then constructs a probability density function to summarize his expectations about next period's market price P_{t+1} using all available information. From this p.d.f., he determines his stock level S_t. Any inventory holding cost $h(S_t)$ is assumed to be incurred at time t.

Let β be the discount factor employed by this speculator and let $\lambda/2$ be the weighting factor of market risk (variance) in his objective function. Then, the speculator entering the market at time t solves the following problem:

$$\text{(A)} \quad \max_{\{S_i\}_{i=t}^{\infty}} \sum_{i=t}^{\infty} \beta^{i-t+1}\{E[P_{i+1}(S_i - S_{i+1}) - h(S_{i+1})] - \frac{\lambda}{2} \text{Var}\,[P_{i+1}(S_i - S_{i+1}) - h(S_{i+1})]\} + P_t(S_{t-1} - S_t) - h(S_t)$$

where the expectations are taken conditional on his available information, hence they are different operators at different points in time. Speculators are assumed to be risk averse, hence $\lambda > 0$.

Before trying to solve (A), we make two further assumptions:[2]

(1) $\text{Var}\,(P_t|P_{t-1}) = \sigma^2, \ \forall\, P_t, P_{t-1}, \ t = 0, 1, 2, \ldots;$

(2) $h(S) = \frac{c(S-b)^2}{2} + d, \ \forall\, S \geq 0$. Assumption (2) incorporates a convenience yield effect, i.e., at stock level b, we achieve minimum inventory cost. If there is no convenience yield effect, then minimum inventory cost should be achieved at S = 0 which implies b = 0.[3] (See [4]).

III. Optimal Speculative Carryover

To solve problem (A), assume that $\lim_{t\to\infty} S_t = b$ (equivalently, this says in the limit, the speculator will choose a minimum cost inventory stock level), and consider a decision beginning at t = 0. Under some regularity assumptions, we can utilize dynamic programming to solve the speculator's problem. Specifically, assume at time T, the speculator's problem is over and his stock decision is $S_t^* = b$, $\forall\, t \geq T$. Therefore, at time (T-1), his problem is:

$$\text{(A1)} \quad \max_{S_{T-1}} \beta[E\{P_T(S_{T-1} - b)\} - \frac{\lambda}{2} \text{Var}\,\{P_T(S_{T-1} - b)\}] + P_{T-1}(S_{T-2} - S_{T-1}) - \{\frac{c(S_{T-1} - b)^2}{2} + d\}$$

(Note that, at (T-1), P_{T-1} and S_{T-2} are both known.)

The first order condition for (A1) is

$$\beta E_T - \beta\lambda\sigma^2(S_{T-1}^* - b) - P_{T-1} - c(S_{T-1}^* - b) = 0$$

$$\Rightarrow S_{T-1}^* = \frac{\beta E_T - P_{T-1} + bc}{\beta\lambda\sigma^2 + c} + \frac{b\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}$$

where $E_T = EP_T$, the conditional expectation of P_T using information about P_{T-1}, and S_t^* is the optimal choice of S_t, t = 0, 1, ...

Next, at time t = T - 2, his problem becomes

$$\text{(A2)} \quad \max_{S_{T-2}} \beta[E\{P_{T-1}(S_{T-2} - S_{T-1}^*)\} - \frac{\lambda}{2} \text{Var}\{P_{T-1}(S_{T-2} - S_{T-1}^*)\}] + P_{T-2}(S_{T-3} - S_{T-2}) - \{\frac{c(S_{T-2} - b)^2}{2} + d\} + K_0$$

where K_0 is a constant term independent of S_{T-2}.

Since

$$\mathrm{Var}\,\{P_{T-1}(S_{T-2} - S^*_{T-1})\} =$$

$$\mathrm{Var}\,\{P_{T-1}(S_{T-2} - \frac{\beta E_T - P_{T-1} + bc + b\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\} =$$

$$E\{S_{T-2}(P_{T-1} - E_{T-1}) - \frac{bc + b\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}(P_{T-1} - E_{T-1}) - (P_{T-1}\frac{\beta E_T - P_{T-1}}{\beta\lambda\sigma^2 + c} - E[P_{T-1}\frac{\beta E_T - P_{T-1}}{\beta\lambda\sigma^2 + c}])\}^2 =$$

$$\sigma^2 S^2_{T-2} - \frac{2(bc + b\beta\lambda\sigma^2)}{\beta\lambda\sigma^2 + c}\sigma^2\, S_{T-2} - \frac{2S_{T-2}}{\beta\lambda\sigma^2 + c}\cdot \mathrm{Cov}(P_{T-1},\, P_{T-1}(\beta E_T - P_{T-1})) + K_1$$

where K_1 is a constant term independent of S_{T-2} and $E_{T-1} = EP_{T-1}$, therefore the first order condition for (A2) is

$$\beta\{E_{T-1} - \lambda\sigma^2 S^*_{T-2} + \frac{\lambda(bc + b\beta\lambda\sigma^2)\sigma^2}{\beta\lambda\sigma^2 + c} + \frac{\lambda}{\beta\lambda\sigma^2 + c}\,\mathrm{Cov}(P_{T-1},\, P_{T-1}(\beta E_T - P_{T-1}))\} - P_{T-2} - c(S^*_{T-2} - b) = 0$$

$$\Rightarrow (\beta\lambda\sigma^2 + c)S^*_{T-2} = \beta E_{T-1} - P_{T-2} + bc + \frac{bc\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} + \frac{b(\beta\lambda\sigma^2)^2}{\beta\lambda\sigma^2 + c} + \frac{\lambda\beta}{\beta\lambda\sigma^2 + c}\,\mathrm{Cov}(P_{T-1}, P_{T-1}(\beta E_T - P_{T-1}))$$

$$\Rightarrow S^*_{T-2} = \frac{\beta E_{T-1} - P_{T-2}}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c}\{1 + \frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\} + b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^2 + \frac{\lambda\beta}{(\beta\lambda\sigma^2 + c)^2}\mathrm{Cov}(P_{T-1}, P_{T-1}(\beta E_T - P_{T-1}))$$

In general, define $f_0(P_t) = \mathrm{Cov}(P_t, P_t(\beta E_{t+1} - P_t))$, $\forall$ t and $f_k(P_t) = \mathrm{Cov}(P_t, P_t f_{k-1}(P_{t+1}))$, $\forall$ k = 1, 2, . . ., $\forall$ t. Then we can state the following theorem:

<u>Theorem 1</u>

The optimal speculative stock level which solves (A) under the assumption that T is the terminal date is:

$$S^*_t = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c}\sum_{i=0}^{T-t-1}(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^i + b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^{T-t} + \frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2}\sum_{i=0}^{T-t-2}(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i f_i(P_{t+1}), \qquad (1)$$

$\forall$ t = 0, 1, 2, . . ., T-1.

[Proof]

The cases t = T-1, T-2 can be checked easily. Now, assume at time t, S^*_t satisfies eq. (1). Then at time (t-1) the speculator's problem is

$$\text{(A3)}\quad \max_{S_{t-1}}\ \beta[E\{P_t(S_{t-1} - S^*_t)\} - \frac{\lambda}{2}\,\mathrm{Var}\{P_t(S_{t-1} - S^*_t)\}] + P_{t-1}(S_{t-2} - S_{t-1}) - \{\frac{c(S_{t-1} - b)^2}{2} + d\} + K_2$$

where K_2 is a constant term independent of S_{t-1}. From eq. (1), we have

$$\mathrm{Var}\{P_t(S_{t-1} - S^*_t)\}$$

$$= E\{(P_t - E_t)S_{t-1} - \frac{bc}{\beta\lambda\sigma^2 + c}\sum_{i=0}^{T-t-1}(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^i(P_t - E_t) - b(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c})^{T-t}(P_t - E_t) - \frac{1}{\beta\lambda\sigma^2 + c}(P_t(\beta E_{t+1} - P_t) - E[P_t(\beta E_{t+1} - P_t)]) - \frac{\beta\lambda}{\beta\lambda\sigma^2 + c)^2}\sum_{\lambda=0}^{T-t-2}(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c})^i(P_t f_i(P_{t+1}) - E[P_t f_i(P_{t+1})])\}^2$$

$$= \sigma^2 S_{t-1}^2 - \frac{2bc\sigma^2}{\beta\lambda\sigma^2 + c} \sum_{i=1}^{T-t-1} \delta^i S_{t-1} - 2b\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^{T-t} \sigma^2 -$$

$$\frac{2}{\beta\lambda\sigma^2 + c} \mathrm{Cov}(P_t, P_t(\beta E_{t+1} - P_t)) -$$

$$\frac{2\beta\lambda}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} \left(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}\right)^i \mathrm{Cov}(P_t, P_t f_i(P_{t+1}))$$

$$= \sigma^2 S_{t-1}^2 - \frac{2bc\sigma^2}{\beta\lambda\sigma^2 + c} \sum_{i=0}^{T-t-1} \delta^i S_{t-1} - 2b\delta^{T-t}\sigma^2 - \frac{2}{\beta\lambda\sigma^2 + c} f_0(P_t) -$$

$$\frac{2\beta\lambda}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} \left(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}\right)^i f_{i+1}(P_t)$$

where $\delta = \beta\lambda\sigma^2/(\beta\lambda\sigma^2 + c)$. Therefore, the first order condition for (A3) is:

$$\beta\{E_t - \lambda\sigma^2 S_{t-1}^* + \frac{bc\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \sum_{i=0}^{T-t-1} \delta^i + b\lambda\sigma^2\delta^{T-t} + \frac{\lambda}{\beta\lambda\sigma^2 + c} f_0(P_t) +$$

$$\frac{\beta\lambda^2}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} \left(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}\right)^i f_{i+1}(P_t)\} - P_{t-1} - c(S_{t-1}^* - b) = 0$$

$$\Rightarrow S_{t-1}^* = \frac{\beta E_t - P_{t-1}}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c} + bc\frac{\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-1} \delta^i + b\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\delta^{T-t} +$$

$$\frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2}\{f_0(P_t) + \sum_{i=0}^{T-t-2} \left(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}\right)^{i+1} f_{i+1}(P_t)\}$$

$$\Rightarrow S_{t-1}^* = \frac{\beta E_t - P_{t-1}}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c} \sum_{k=0}^{T-t} \delta^K +$$

$$b\delta^{T-t+1} + \frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2} \sum_{k=0}^{T-t-1} \left(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}\right)^K f_K(P_t),$$

by letting $K = i + 1$. Therefore, the proof is completed.

Q.E.D.

Eq. (1) expresses the optimal speculative stock level as the summation of four terms: (1) the current expected profit effect $\frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c}$; (2) the terminal convenience yield effect $b\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^{T-t}$;

(3) the cost-factor-and-convenience-yield interaction effect

$$\frac{bc}{\beta\lambda\sigma^2 + c} \sum_{i=0}^{T-t-1} \left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^i$$

and (4) the covariance risk effect

$$\frac{\beta\lambda}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} \left(\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}\right)^i f_i(P_{t+1}).$$

Among these four effects, (2) vanishes as T approaches infinity, while (3) and (4) are special features arising because the mean-variance approach is used to describe the speculator's preferences. These will be discussed further in the following section.

IV. Properties of the Optimal Stock Level

First, note that in the derivations leading to eq. (1), we implicitly assumed $S_t^* \geq 0$. However, since short selling is not allowed, the optimal speculative stock level should be written as $\hat{S}_t = \max(S_t^*, 0)$ for every $t > 0$; and if we have t', such that $S_{t'}^* < 0$, then all formulas for S_t^*, $t \leq t'$ now are invalid. This introduces a complex discontinuity into the problem. In the general case, we will simply assume $S_t^* \geq 0$. [There are some special cases, however, in which $S_t^* \geq 0$ can be proved (i.e., the case where $c \equiv 0$)].

Second, assume $\lambda = 0$ and let T approach infinity. Then eq. (1) reduces to the case considered in [4], i.e., all competitive speculators are expected-profit maximizing agents. Hence, eq. (1) becomes $S_t^* = \frac{\beta E_{t+1} - P_t}{c} + b$ since the terminal convenience yield effect (2) and covariance risk effect (4) both vanish when $\lambda = 0$,

$T \to \infty$, and the interaction effect becomes b. There is one period time-lag difference between our model and that used in [4] as to when the inventory cost occurs. Adjusting for this, we obtain the optimal stock level derived in [4], which is therefore a special case of our model.

Third, note that eq. (1) holds when T is the terminal date. But to solve (A), we must let T approach infinity which creates convergence problems. Note that if $c > 0$, then $0 < \delta = \frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1$, and convergence problems only arise from the covariance risk effect. However, if $c = 0$ (i.e., there are no variable inventory costs), then all the terms except (2) require further consideration.

The last points we want to make are about the interaction effect and the covariance risk effect. Each of these is a discounted sum of a sequence but using apparently different discount rates, $\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}$ and $\frac{\beta\lambda}{\beta\lambda\sigma^2 + c}$ respectively. This is somewhat misleading. When we introduce $f_K(\cdot)$ into eq. (1), it turns out that both expressions involve the same discount rate $\delta = \frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}$. If we let $b = 0$ or $c = 0$, then the interaction effect vanishes (but the covariance risk effect remains). As for $f_K(\cdot)$, these functions all take the covariance operator form. For example, $f_0(P_t) = \mathrm{Cov}(P_t, P_t(\beta E_{t+1} - P_t))$ is the covariance between price and expected profit for the next period;

$$f_K(P_t) = \mathrm{Cov}(P_t, P_t f_{K-1}(P_{t+1}))$$

$$= \mathrm{Cov}(P_t, P_t\mathrm{Cov}(P_{t+1}, P_{t+1}f_{K-2}(P_{t+2})) = \ldots$$
$$= \mathrm{Cov}(P_t, P_t\mathrm{Cov}(P_{t+1}, P_{t+1}\mathrm{Cov}(P_{t+2}, \ldots,$$
$$\mathrm{Cov}(P_{t+K}, P_{t+K}(\beta E_{t+K+1} - P_{t+K}))$$

measures the covariance between price and expected profit K periods later (by updating information at each subsequential future period). Therefore, we named (4) as the covariance risk effect. Note that this effect comes across time, rather than across alternatives at a point in time (which leads to a covariance risk effect in the Capital Asset Pricing Models).

V. Linear Expectation Rule

Now, assume every speculator is identical with price expectation formation equation given by:

$$P_t^e = \delta + \alpha P_{t-1} + \varepsilon_t, \quad \forall\, t \tag{2}$$

where α is the price expectation adjustment coefficient, $\delta/(1 - \alpha)$ is the long-run rational expectations equilibrium price, $\{\varepsilon_t\}$ is a sequence of identically independently distributed random variables with $E(\varepsilon_t|P_{t-1}) = 0$, $\mathrm{Var}(\varepsilon_t|P_{t-1}) = \sigma^2$ and $E(\varepsilon_t^3|P_{t-1}) = 0$ (i.e., the probability density function of ε_t is symmetric with respect to zero), $\forall\, t$. When $\alpha > 1$, we say the speculator is responsive; when $\alpha < 1$, we say he is unresponsive.

Using (2), we can determine $f_K(P_t)$ for every $K \geq 0$. For example,

$$f_0(P_t) = \mathrm{Cov}(P_t, P_t(\beta E_{t+1} - P_t)) = \mathrm{Cov}(P_t, P_t(\beta\delta + (\beta\alpha - 1)P_t))$$

$$= \beta\delta\sigma^2 + (\beta\alpha - 1)\ \mathrm{Cov}(P_t, P_t^2)$$
$$= \beta\delta\sigma^2 + (\beta\alpha - 1)E\{\varepsilon_t \cdot [2(\delta + \alpha P_{t-1})\varepsilon_t + \varepsilon_t^2 - \sigma^2]\}$$
$$= \{\beta\delta + 2(\beta\alpha - 1)(\delta + \alpha P_{t-1})\}\sigma^2$$

also,

$$f_1(P_t) = \mathrm{Cov}(P_t, P_t f_0(P_{t+1})) = \mathrm{Cov}(P_t, P_t(\beta\delta + 2(\beta\alpha - 1)(\delta + \alpha P_t))\sigma^2)$$
$$= \sigma^4\{\beta\delta + 2(\beta\alpha - 1)\delta\} + 2\sigma^2(\beta\alpha - 1)\alpha\ \mathrm{Cov}(P_t, P_t^2)$$
$$= \{\beta\delta + 2(\beta\alpha - 1)\delta + 2\alpha(\beta\alpha - 1)(\delta + \alpha P_{t-1})\}\sigma^4$$

In general, we can prove the following theorem:

<u>Theorem 2</u>

Under the linear expectation rule (eq. (2)),

$$f_k(P_t) = \{\beta\delta + 2\delta(\beta\alpha - 1)\sum_{j=1}^{K+1}\alpha^{j-1} + 2\alpha^{K+1}(\beta\alpha - 1)P_{t-1}\}\sigma^{2K+2}, \quad \forall\ K, \quad \forall\ P_t. \tag{3}$$

[Proof]

The cases where K = 0 and K = 1 can be easily checked. Now, assume for $K = i$, $f_k(P_t)$ satisfies eq. (3), hence

$$f_{i+1}(P_t) = \mathrm{Cov}(P_t, P_t f_i(P_{t+1}))$$
$$= \mathrm{Cov}(P_t, P_t(\beta\delta + \delta\sum_{j=1}^{i+1}2(\beta\alpha - 1)\alpha^{j-1} + 2\alpha^{i+1}(\beta\alpha - 1)P_t)\sigma^{2i+2})$$
$$= \{\beta\delta + \delta\sum_{j=1}^{i+1}2(\beta\alpha - 1)\alpha^{j-1}\}\sigma^{2i+4} + 2\alpha^{i+1}(\beta\alpha - 1)\sigma^{2i+2}\mathrm{Cov}(P_t, P_t^2)$$
$$= \{\beta\delta + \delta\sum_{j=1}^{i+1}2(\beta\alpha - 1)\alpha^{j-1}\}\sigma^{2i+4} + 2(\beta\alpha - 1)\alpha^{i+1}\sigma^{2i+4}(\delta + \alpha P_{t-1})$$

$$= \{\beta\delta + 2\delta(\beta\alpha - 1)\sum_{j=1}^{i+2}\alpha^{j-1} + 2(\beta\alpha - 1)\alpha^{i+2}P_{t-1}\}\sigma^{2i+4},$$

which completes the proof.

Q.E.D.

Substituting eq. (3) into eq. (1), we have

$$S_t^* = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c} + \frac{bc}{\beta\lambda\sigma^2 + c}\sum_{i=0}^{T-t-1}\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^i + b\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^{T-t} + \frac{\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2}\sum_{i=0}^{T-t-2}\left(\frac{\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c}\right)^i \cdot$$
$$\{\beta\delta + 2\delta(\beta\alpha - 1)\sum_{j=1}^{i+1}\alpha^{j-1} + 2\alpha^{i+1}(\beta\alpha - 1)P_t\}. \tag{4}$$

The next problem we consider is conditions under which S_t^* will converge as $T \to \infty$.

<u>VI. Zero Variable Inventory Cost</u>

When c = 0, eq. (4) becomes

$$S_t^* = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2} + b + \sum_{i=0}^{T-t-2}\left\{\frac{\beta\delta}{\beta\lambda\sigma^2} + \frac{2\delta(\beta\alpha - 1)}{\beta\lambda\sigma^2}\sum_{j=1}^{i+1}\alpha^{j-1} + \frac{2\alpha^{i+1}(\beta\alpha -)P_t}{\beta\lambda\sigma^2}\right\}$$

$$
= \begin{cases} \dfrac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2} + b + \sum_{i=0}^{T-t-2}\{\dfrac{\beta\delta}{\beta\lambda\sigma^2} + \dfrac{2\delta(\beta\alpha - 1)}{\beta\lambda\sigma^2} \cdot \dfrac{1 - \alpha^{i+1}}{1 - \alpha} + \dfrac{2\alpha^{i+1}(\beta\alpha - 1)P_t}{\beta\lambda\sigma^2}\}, & \text{if } \alpha \neq 1 \\ \dfrac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2} + b + \sum_{i=0}^{T-t-2}\{\dfrac{\beta\delta}{\beta\lambda\sigma^2} + \dfrac{2\delta(\beta - 1)(i + 1)}{\beta\lambda\sigma^2} + \dfrac{2(\beta - 1)P_t}{\beta\lambda\sigma^2}\}, & \text{if } \alpha = 1 \end{cases} \tag{4'}
$$

Theorem 3

Given $c = 0$, assume $\delta \neq 0$. Then if $T \to \infty$, S_t^* is unbounded for every t.

[Proof]

Obviously, when $\alpha = 1$, $\delta \neq 0$, $T \to \infty$, then $S_t^* \to -\infty$. On the other hand, if $\alpha \neq 1$, then for S_t^* to be bounded, we must require that: (1) $\alpha < 1$ and (2) $\lim_{i\to\infty} \dfrac{\beta\delta}{\beta\lambda\sigma^2} + \dfrac{2\delta(\beta\alpha - 1)}{\beta\lambda\sigma^2} \cdot \dfrac{1 - \alpha^{i+1}}{1 - \alpha} = 0$. Now, (2) implies $\beta(1 - \alpha) + 2(\beta\alpha - 1) = 0 \Rightarrow \beta(\alpha + 1) = 2$, contradicting $\alpha, \beta < 1$. Hence S_t^* is unbounded when $\delta \neq 0$, $T \to \infty$.

Q.E.D.

Although S_t^* is unbounded from below, yet since short selling is prohibited, S_t must be non-negative. Hence the optimal stock $\hat{S}_t = \max(S_t^*, 0)$ is either ∞ or 0 when $\delta \neq 0$ and $T \to \infty$. This implies

Corollary 1

Given $c = 0$, $0 \leq \hat{S}_t < \infty$, $\forall$ t and $\hat{S}_t > 0$ for some t implies one of the following conditions:

(i) $\delta = 0$, $\alpha < 1$

(ii) T is finite.

Corollary 2

Given $c = 0$, $\delta = 0$, $\alpha < 1$, and $T \to \infty$ implies $\hat{S}_t = 0$, $\forall$ t.

Corollary 1 and 2 show that with zero variable inventory holding costs, when $T \to \infty$, and short selling is prohibited, then the speculator either accumulates unbounded stocks or no stocks at all, i.e., speculators are either highly active or totally inactive. For example, when $\delta = 0$, $\alpha < 1$, they always hold zero stock. In the other cases, when $\delta \neq 0$, they might switch from an unbounded stock to zero at some points, and then remain for few periods, finally they switch from zero to an unbounded level of stocks. This implies they are highly active.

Theorem 4

Given $c = 0$, when $T < \infty$, any time-independent linear expectation of speculators won't be fulfilled.

The proof of Theorem 4 involves the structure of non-speculative excess demand, therefore we'll put it into the Appendix after the introductions of market demand structure. Nonetheless, the reason we state Theorem 4 here is to claim that "$T < \infty$" is also not a useful assumption to avoid the "unboundedness" problems that arise when $c = 0$. In the following sections, $c \neq 0$ is assumed.

VII. Properties of the Optimal Stock Level

When $c \neq 0$, eq. (4) can be written as:

$$S_t^* = \frac{\beta E_{t+1} - P_t}{\beta\lambda\sigma^2 + c} + M_{1t} + M_{2t} + M_{3t} + M_{4t} + M_{5t}$$

where

$$M_{1t} = \frac{bc}{\beta\lambda\sigma^2 + c} \cdot \frac{1 - \phi^{T-t}}{1 - \phi},$$

$$M_{2t} = b\phi^{T-t}$$

$$M_{3t} = \frac{\beta^2\delta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \cdot \frac{1 - \phi^{T-t-1}}{1 - \phi}.$$

$$M_{4t} = \begin{cases} \dfrac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2} \displaystyle\sum_{i=0}^{T-t-2} \phi^i \cdot \frac{1 - \alpha^i}{1 - \alpha}, \text{ where } \alpha \neq 1 \\ \dfrac{2\delta(\beta - 1)\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)^2} \displaystyle\sum_{i=0}^{T-t-2} \phi^i(i + 1), \text{ when } \alpha = 1 \end{cases}$$

$$M_{5t} = \frac{2\beta\lambda\sigma^2(\beta\alpha - 1)P_t}{(\beta\lambda\sigma^2 + c)^2} \sum_{i=0}^{T-t-2} \alpha^{i+1}\phi^i$$

and $\quad \phi = \dfrac{\beta\lambda\sigma^2}{(\beta\lambda\sigma^2 + c)}$

Hence, as $T \to \infty$, $M_{1t} \to \dfrac{bc}{\beta\lambda\sigma^2 + c} \cdot \dfrac{1}{1 - \phi} = b$; $M_{2t} \to 0$;

$$M_{3t} \to \frac{\beta^2\delta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \cdot \frac{1}{1 - \phi} = \frac{\beta^2\delta\lambda\sigma^2}{c(\beta\lambda\sigma^2 + c)}$$

and

$$M_{5t} \to \begin{cases} \dfrac{2\alpha\beta\lambda\sigma^2(\beta\alpha - 1)P_t}{(\beta\lambda\sigma^2 + c)[(1 - \alpha)\beta\lambda\sigma^2 + c]}, \text{ when } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1 \\ \infty, \text{ when } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \geq 1 \end{cases}$$

$$M_{4t} \to \begin{cases} \dfrac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{c(1 - \alpha)(\beta\lambda\sigma^2 + c)} - \dfrac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{(1 - \alpha)(\beta\lambda\sigma^2 + c)[(1 - \alpha)\beta\lambda\sigma^2 + c]}, \\ \qquad \text{when } \alpha \neq 1 \text{ and } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1 \\ \pm\infty \text{ when } \alpha \neq 1 \text{ and } \dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} \geq 1 \\ \dfrac{2\delta(\beta - 1)\beta\lambda\sigma^2}{c^2} \text{ when } \alpha = 1 \end{cases}$$

Note that when $\alpha = 1$, $\dfrac{\alpha\beta\lambda\sigma^2}{\beta\lambda\sigma^2 + c} < 1$ is satisfied. Therefore, we have the following theorem:

Theorem 5

Assume $T \to \infty$. If $\alpha\beta\lambda\sigma^2 \geq \beta\lambda\sigma^2 + c$, then S_t^* is unbounded. Furthermore, whether $S_t^* = \infty$ or $-\infty$ depends on M_{4t} and M_{5t}. On the other hand, if $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$, then as $T \to \infty$, S_t^* will converge to

$$\bar{S}_t = M + \frac{(\beta\alpha - 1)P_t}{\beta\lambda\sigma^2 + c}\left[\frac{(1 + \alpha)\beta\lambda\sigma^2 + c}{(1 - \alpha)\beta\lambda\sigma^2 + c}\right]$$

where

$$M = \frac{\beta\delta}{\beta\lambda\sigma^2 + c} + \frac{\beta^2\delta\lambda\sigma^2}{c(\beta\lambda\sigma^2 + c)} + b +$$

$$\frac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{c(1 - \alpha)(\beta\lambda\sigma^2 + c)} - \frac{2\delta(\beta\alpha - 1)\beta\lambda\sigma^2}{c(1 - \alpha)(\beta\lambda\sigma^2 + c)[(1 - \alpha)\beta\lambda\sigma^2 + c]},$$
when $\alpha \neq 1$;

and

$$M = \frac{\beta\delta}{\beta\lambda\sigma^2 + c} + \frac{\beta^2\delta\lambda\sigma^2}{c(\beta\lambda\sigma^2 + c)} + b + \frac{2\delta(\beta - 1)\beta\lambda\sigma^2}{c^2},$$
when $\alpha = 1$.

<u>Corollary 3</u>

Given $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$, $\bar{S}_t > 0$ implies

(i) $\partial\bar{S}_t/\partial b = 1$, $\forall$ t.

(ii) $\text{sgn}(\partial\bar{S}_t/\partial P_t) = \text{sgn}(\beta\alpha - 1)$, $\forall$ t.

[Proof]

(i) is obvious. For (ii), if $\alpha < 1$, then $\text{sgn}(\partial\bar{S}_t/\partial P_t) = \text{sgn}(\beta\alpha - 1) < 0$, since $1 - \alpha > 0$ and $\beta, \alpha, \sigma^2, c > 0$. If $\alpha > 1$, then since $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c \Rightarrow c + (1 - \alpha)\beta\lambda\sigma^2 > 0$, hence $\text{sgn}(\partial\bar{S}_t/\partial P_t) = \text{sgn}(\beta\alpha - 1)$, $\forall$ t.

Q.E.D.

Theorem 5 shows that $\{\bar{S}_t\}$ is the solution for problem (A) when $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$ and $c > 0$ (note that, by hypothesis, $\bar{S}_t \geq 0$, $\forall$ t), therefore the optimal speculative stock level is fully characterized. Otherwise, we always have $\bar{S}_t = \infty$ or 0. Corollary 3 shows that as the minimum-cost stock level b changes by one unit, the optimal stock level $\bar{S}_t$ also changes by one unit in the same direction for every t. Furthermore, when the current price P_t changes, which direction $\bar{S}_t$

will change is determined by the sign of $(\beta\alpha - 1)$.

<u>VIII. Market Price Behavior</u>

Now, since we take the behavior of non-speculators as given, we can summarize their impacts on the market by a non- speculative excess demand function. Following [4], we postulate a linear non-speculative excess demand function of the form:

$$D_t = -aP_t + \gamma_t, \; a > 0 \tag{5}$$

where $\{\gamma_t\}$ is a sequence of identically independently distributed random variables with $E(\gamma_t) = \mu$, $Var(\gamma_t) = V$.

By the market clearing condition,[4] we have

$$\bar{S}_{t-1} - \bar{S}_t = -aP_t + \gamma_t, \; \forall \; t$$

$$\Rightarrow zP_{t-1} - zP_t = -aP_t + \gamma_t, \; \forall \; t$$

$$\Rightarrow P_t = \frac{z}{z - a}P_{t-1} - \frac{\gamma_t}{z - a}, \; \forall \; t \tag{6}$$

$$\Rightarrow P_t = \left(\frac{z}{z - a}\right)^t P_0 - \sum_{j=0}^{t-1}\left(\frac{z}{z - a}\right)^j \frac{\gamma_{t-j}}{z - a}, \; \forall \; t = 0, 1, 2, \ldots \tag{7}$$

where $z = \frac{(\beta\alpha - 1)}{\beta\lambda\sigma^2 + c}\left\{\frac{(1 + \alpha)\beta\lambda\sigma^2 + c}{(1 - \alpha)\beta\lambda\sigma^2 + c}\right\}$. Note that this result is derived when $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$ and $0 \leq \bar{S}_t < \infty$, $\forall$ t.

From (7), we have

$$EP_t = \left(\frac{z}{z - a}\right)^t P_0 - \sum_{j=0}^{t-1}\left(\frac{z}{z - a}\right)^j \frac{\mu}{z - a}$$

$$= \left(\frac{z}{z - a}\right)^t P_0 - \frac{\mu}{z - a} \cdot \frac{1 - \left(\frac{z}{z - a}\right)^t}{1 - \frac{z}{z - a}}$$

$$= w^t P_0 + \frac{\mu}{a}(1 - w^t), \text{ where } w = \frac{z}{z - a};$$

$$\text{Var } P_t = \text{Var}\{(\frac{z}{z - a})^t P_0 - \sum_{j=0}^{t-1}(\frac{z}{z - a})^j \frac{\gamma_{t-j}}{z - a}\}$$

$$= \frac{V}{(z - a)^2}\sum_{j=0}^{t-1}(\frac{z}{z - a})^{2j} = \frac{(1 - w)^2 V}{a^2} \cdot \frac{1 - w^{2t}}{1 - w^2}.$$

Since $w = \frac{z}{z - a} \Rightarrow za - aw = z \Rightarrow z = \frac{aw}{w - 1} \Rightarrow z - a = \frac{a}{w - 1}$,

$$\text{Cov}(P_t, P_{t-h}) = \text{Cov}(-\sum_{j=0}^{t-1} w^j \frac{\gamma_{t-j}}{z - a}, -\sum_{k=0}^{t-h-1} w^k \frac{\gamma_{t-h-k}}{z - a})$$

$$= \sum_{k=0}^{t-h-1} w^{2k+h} \cdot \frac{V}{(z - a)^2} = \frac{(1 - w)^2 V}{a^2} \cdot w^h \cdot \frac{1 - w^{2t-2h}}{1 - w^2}, \text{ when } 0 \le h < t$$

This proves:

<u>Theorem 6</u>

Given $c \neq 0$, $\alpha\beta\lambda\sigma^2 < \beta\lambda\sigma^2 + c$, $0 \le \bar{S}_t < \infty \ \forall\, t$,

(i) $$\lim_{t \to \infty} EP_t = \begin{cases} \frac{\mu}{a}, & \text{if } |w| < 1 \\ \pm\infty, & \text{if } |w| > 1 \end{cases}$$

(ii) $$\lim_{t \to \infty} \text{Var}P_t = \begin{cases} \frac{(1 - w)V}{a^2(1 + w)}, & \text{if } |w| < 1 \\ \infty, & \text{if } |w| > 1 \end{cases}$$

(iii) $$\lim_{t \to \infty} \text{Cov}(P_t, P_{t-h}) = \begin{cases} \frac{w^h(1 - w)V}{a^2(1 + w)}, & \text{if } |w| < 1 \\ \pm\infty, & \text{if } |w| > 1 \end{cases}$$

Since we want price to be non-negative, we make the following assumptions: (i) $w > 0$ and (ii) $P_0 > \frac{\mu}{a}$. Therefore, for $|w| < 1$, we need $0 < \frac{z}{z - a} < 1 \Rightarrow z < 0 \Rightarrow \beta\alpha < 1$, since $(1 - \alpha)\beta\lambda\sigma^2 + c > 0$. Now, if $c > (1 - \beta)\lambda\sigma^2$, then $\beta\lambda\sigma^2 + c > \lambda\sigma^2 \Rightarrow 1 + \frac{c}{\beta\lambda\sigma^2} > \frac{1}{\beta}$, hence $\alpha < \frac{\beta\lambda\sigma^2 + c}{\beta\lambda\sigma^2} \not\Leftrightarrow \alpha < \frac{1}{\beta}$ which establishes the following:

<u>Theorem 7</u>

Assume $\alpha > 1$. If $c > (1 - \beta)\lambda\sigma^2$, then S_t^* bounded and $\lim_{t \to \infty} EP_t$, $\lim_{t \to \infty} \text{Var}P_t$ unbounded do not violate market clearing. Under this configuration, the action of competitive speculators will destabilize prices.

As to whether the speculator's expectations will be fulfilled, we can compare eq. (6) and eq. (2) to derive the following theorem:

<u>Theorem 8</u>

Fulfilling of speculator's expectation implies:

(i) $\frac{z}{z - a} = \alpha$ and (ii) $\frac{\gamma_t}{a - z} = \delta + \varepsilon_t$ where

$$z = \frac{\beta\alpha - 1}{\beta\lambda\sigma^2 + c}\{\frac{(1 + \alpha)\beta\lambda\sigma^2 + c}{(1 - \alpha)\beta\lambda\sigma^2 + c}\},$$

and (ii) holds for every $t = 1, 2, \ldots$.

Corollary 4

If speculator's expectations are fulfilled, then (i) $\alpha < 1$, (ii) $\mu = \delta(a - z)$, (iii) $V^2 = (a - z)^2\sigma^2$.

[Proof]

Assume $\alpha = 1$, then fulfilling expectation implied $\frac{z}{z-a} = 1$ $\Rightarrow z = z - a \Rightarrow a = 0$, contradiction. On the other hand, if $\alpha > 1$, then $\frac{z}{(z-a)} = \alpha \Rightarrow z > a > 0$ and $\frac{\partial \bar{S}_t}{\partial P_t} = z > a$. Therefore, as $P_t \to \infty$, $\bar{S}_t \to \infty$ which is unbounded. Since we only dealt with bounded $\bar{S}_t$, hence $\alpha < 1$ is required. (ii) and (iii) are derived from $E(\frac{\gamma_t}{a - z}) = E(\delta + \varepsilon_t)$ and $Var(\frac{\gamma_t}{a - z}) = Var(\delta + \varepsilon_t)$, respectively (where expectations are conditional on available information).

Q.E.D.

Therefore, when speculator's expectations are fulfilled, EP_t, $VarP_t$ and $Cov(P_t, P_{t-h})$ are all bounded. Also, S_t^* is bounded.

IV. Profitable Speculation

In this section, we turn to Friedman's conjecture, i.e., profitable speculation necessarily stabilizes prices. Recall taht, in problem (A), $S_t = b$, $\forall$ t is a feasible strategy, therefore, any strategy $\{S_t\}$ with $S_t \neq b$ for some t certainly incurs positive profits

(actually, the profits must be high enough to cover the losses from change in variances). From Theorem 5, this implies $M + zP_t \neq b$ for some t which is easily to be satisfied.

Now, from Theorem 7, when $\alpha > 1$, there would be destabilizing profitable speculation. However, in this case, the speculator's expectations won't be fulfilled. On the other hand, when their expectations are fulfilled, $\alpha < 1$ and

$$VarP_t = \frac{(1 - w)V}{a^2(1 + w)} \cdot (1 - w^{2t}) < \frac{V}{a^2}$$

(since $\alpha < 1 \Rightarrow z < 0 \Rightarrow 0 < w = \frac{z}{z - a} < 1$), where $\frac{V}{a^2} = VarP_t$ when there are no speculators. Therefore,

Theorem 9

At a rational expectations equilibrium (i.e., speculators' expectations are fulfilled), and given a linear non-speculative excess demand, profitable speculation always stabilizes prices.

Theorem 9 leaves it open whether at a rational expectations equilibrium with non-linear non-speculative excess demand, profitable speculation always stabilizes prices. Because of earlier results (see [1], [3], [6]), it seems unlikely that Friedman's conjecture will hold with non-linear excess demands, however.

V. Conclusion

In this paper, speculators are taken to be risk averse, and a mean-variance approach was employed. Under this approach, the optimal stock level for speculators has been derived. Nonetheless, this stock level might be unbounded. To carry the analysis further, we found when marginal inventory cost is zero, speculators are either hightly active ($S_t = \infty$) or inactive ($S_t = 0$). To resolve the problem of unboundedness of S_t when c = 0 requires either the assumption that the long-run equilibrium price equals zero (which leads to $S_t = 0, \ \forall t$) or the assumption of finite horizon (in which case speculators' expectations won't be fulfilled[5]).

On the other hand, when the inventory carrying cost function is of a non-degenerate quadratic form, one possible equilibrium configuration involves bounded stock levels and unbounded prices, with the expectation adjustment coefficient greater than 1. However, this won't constitute a rational expecations equilibrium.[6] When a rational expectations equili exists given linear non-speculative excess demand, the stock level is bounded, price is also bounded, and Friedman's conjecture is verified, i.e., profitable speculation necessarily stabilizes prices.

Appendix: Proof of Theorem 4

By inspecting eq. (2) and eq. (4'), we know that unless $\beta\alpha - 1 = 0$, speculator's expectations won't be fulfilled, since in (4'), price terms involve multiplicative time factors when $\beta\alpha \neq 1$. On the other hand, when $\beta\alpha - 1 = 0$, then

$$S_t^* = \frac{\beta\delta}{\beta\lambda\sigma^2} + b + \frac{\beta\delta}{\beta\lambda\sigma^2}(T - t - 1) = \frac{\beta\delta}{\beta\lambda\sigma^2}(T - t) + b \geq 0, \ \forall t \leq T$$

$$\Rightarrow S_{t-1}^* - S_t^* = \frac{\delta}{\lambda\sigma^2}, \ \forall t \leq T$$

Therefore, the market clearing condition becomes

$$- aP_t + \gamma_t = \frac{\delta}{\lambda\sigma^2}, \ \forall t \leq T$$

$$\Rightarrow P_t = \frac{\gamma_t}{a} - \frac{\delta}{a\lambda\sigma^2}, \ \forall t \leq T$$

Now, for expectations to be fulfilled, we need $\alpha = 0$ which contradicts $\beta\alpha - 1 = 0$. Hence, the proof is completed.

Footnotes

* I am indebted to James Quirk for helpful discussions and editings, also to Richard McKelvey for comments on earlier drafts. All errors, of course, remain mine.

1. On the other hand, both Sarris [5] and Turnovsky [7] employed mean-variance approach to determine one-period optimal stock level, without taking account the dynamic effects.

2. The assumption $\text{Var}(P_t | P_{t-1}) = \sigma^2$, $\forall$ t can be relaxed to $\text{Var}(P_t | P_{t-1}) = \sigma_t^2$ which is a constant term independent of P_t, P_{t-1}, but might change over time. Under this assumption, the results can be easily adjusted to characterize the optimal stock level. Nonetheless, the market price process will be highly complexized and difficult to proceed.

3. Note that, suppose instead of quadatic form, we use linear form for inventory cost function. Thus, when $b = 0$ (i.e., no convenience yield), the inventory cost curve is a straight line over $[0,\infty)$. However, if $b \neq 0$, then we have to introduce a kinked point in inventory cost curve.

4. Strictly speaking, the left-hand side of market clearing equation should be multiplied by the number of representative speculators, but without loss of generality, we set this factor to be one.

5. In the one-period framework, considering all the agents in the

market, Turnovsky [7] showed that constant marginal inventory cost may lead to nonexistence of rational expectations equilibrium in futures market as well.

6. Without considering the "unboundedness" problem of optimal stock level, when $\alpha > 1$ and the speculator's expectation is fulfilled, the optimal stock level will increase monotonically over time. Therefore, there will never be realized profits.

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